MA 345 Differential Equations Dr. E. Jacobs Exam I Practice

Note: This exam was given in Fall 2024

1 - 3. (60 points) Find the solution of each of the following differential equation. Make sure you solve for y in terms of x. Don't forget about the given initial conditions.

$$\frac{dy}{dx} = \frac{2y^2}{(x+1)^3} \qquad \text{where } y(0) = 1$$

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2.
$$\frac{dy}{dx} + \frac{1}{2x}y = \frac{1}{\sqrt{x}}$$
 where $y(1) = 0$

3.
$$(2y - 3x) dx + x dy = 0$$
 where $y(1) = 1$

4. (20 points) Smoke is blowing through a broken window into a room. The air coming into the room is coming in at 8 cubic meters/hour with a concentration of smoke particles of 100 milligrams/cubic meter. At the same time, an exhaust fan on the other side of the room is removing air at the rate of 8 cubic meters/hour.

The volume of the room is 32 cubic meters.



Let y(t) be the mass (in milligrams) of smoke particles in the room at time t hours.

a) Using the $\binom{\text{rate}}{\text{in}} - \binom{\text{rate}}{\text{out}}$ principle, write a differential equation that correctly determines y as a function of t.

b) Solve the differential equation. Assume y(0) = 0

5. A 200 kilogram sailboat is floating motionless in the water. Suddenly, a wind with a constant force of 50 newtons begins to push the boat forward. The drag force, due to water resistance, is $F_R = 100v$ and acts in the opposite direction of the motion of the boat.



Let v(t) be the velocity of the sailboat after t sec. Use fact that (mass)(acceleration) = Net Force, write an appropriate differential equation that determines v(t)

| a) | $\frac{dv}{dt} = \frac{1}{4} - \frac{1}{2}v$ | b) | $\frac{dv}{dt} = \frac{1}{2} - \frac{1}{4}v$ |
|----|--|----|--|
| c) | $\frac{dv}{dt} = \frac{1}{4} + \frac{1}{2}v$ | d) | $\frac{dv}{dt} = \frac{1}{2} + \frac{1}{4}v$ |

6. A differential equation is called an *exact differen*tial equation if it can be written in the form dF = 0. Only one of the following equations is exact. Which one?

a)
$$(x+2y)dx + (3x+4y)dy = 0$$

b) $(3x+2y)dx + (2x+4y)dy = 0$
c) $(2x+3y)dx + (4x+2y)dy = 0$

d) (2x+3y) dx + (x+2y) dy = 0

7. Which of the following would be an integrating factor μ for the differential equation

$$\frac{dy}{dx} + \left(2x + \frac{1}{x}\right)y = x^2$$

a) $\mu = \ln x$ b) $\mu = x e^{x^2}$ c) $\mu = x^2 + \ln x$ d) $\mu = x^2 e^{x^2}$ 8. The equation $(y^2 + 4xy) dx + (2xy + 2x^2) dy = 0$ can be written in the form dF = 0. Which of the following expressions is F?

a) $F = 2x^2y^2 + x^2y + C$ b) $F = 2x^2 + y^2 + C$

c) $F = 2xy^2 + x^2y + C$ d) $F = xy^2 + 2x^2y + C$