MA 345 **Practice Exam III** Dr. E. Jacobs

Part I - Multiple Choice Section

1. The solution of the equation $(D^2 - 1)y = x^2$ has the form $y = y_h + y_p$, where y_h is then *homogeneous* solution and y_p is a *particular* solution. Which of the following is the general form of the particular solution.

a) $y_p = a_1e^x + a_2e^{-x}$ **b)** $y_p = a_1 + a_2x + a_3x^2$ **c)** $y_p = a_1e^x + a_2e^{-x} + a_3x^2$
d) $y_p = a_1x + a_2x^2 + a_3x^3$ **e)** $y_p = a_1e^x + a_2e^{-x} + a_3x + a_4x^2$

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2. The solution of the equation $(D^2 - D)y = 1 + e^x$ has the form $y = y_h + y_p$, where y_h is then *homogeneous* solution and y_p is a *particular* solution. Which of the following is the general form of the particular solution.

a) $y_p = a_1x^2 + a_2e^x$ **b)** $y_p = a_1x^2e^x$ **c)** $y_p = a_1 + a_2x + a_3e^x$
d) $y_p = a_1 + a_2xe^x$ **e)** $y_p = a_1x + a_2xe^x$

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3. The particular solution of the equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} + 4y = 2\cos 2x - 4\sin 2x$ has a particular solution with the general form $y_p = A\cos 2x + B\sin 2x$. Calculate the coefficient A.

a) $A = 0$ **b)** $A = 1$ **c)** $A = -1$ **d)** $A = 2$ **e)** $A = -2$

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4. What is the annihilator of $xe^x + x$?

a) $(D-1)^2$ **b)** $2D^2 - 1$ **c)** $(D^2-1)^2$ **d)** $D^2(D-1)^2$ **e)** $D^2(D^2-1)$

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5. Find the inverse Laplace transform of the following expression:

$$\frac{s}{(s+1)(s+2)}$$

a) $e^{-2t} + e^{-t}$ **b)** $e^{-2t} - 2e^{-t}$ **c)** $2e^{-2t} - e^{-t}$
d) $2e^{2t} - e^t$ **e)** e^{-3t}

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6. Find the inverse Laplace transform of the following expression:

$$\frac{s}{s^2 - 4s + 5}$$

a) $e^{2t}(\cos t + 2\sin t)$ **b)** $e^t(\cos 2t + \sin 2t)$ **c)** $e^{-2t}(2\cos t + \sin t)$
d) $e^{2t}\cos t$ **e)** $t + e^{-2t}\sin t$

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7. Suppose $y = y(t)$ is the solution of the following initial value problem:

$$y'' - 4y' + 4y = 0 \qquad \text{where } y(0) = 1 \text{ and } y'(0) = 2$$

Which of the following would equal $\mathcal{L}(y)$?

a) $\frac{1}{(s-2)^2}$ **b)** $\frac{s}{(s-1)^2}$ **c)** $\frac{1}{s-2}$ **d)** $\frac{1}{s+2}$ **e)** $\frac{s+1}{(s-1)^2}$

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8. Use the Exponential Shift Theorem to simplify the following:

$$(D-2)^4\left(\left(x^3+\cos x\right)e^{2x}\right)$$

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9. Find the general solution of the following equation. Show all work.

$$(D+1)^2 y = e^{-x}$$

10.

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = \frac{3}{\sqrt{x}}e^x$$

a) If we were to use the method of variation of parameters to solve this problem, we would need to calculate a determinant called the *Wronskian.* Calculate the Wronskian for this problem.

b) Solve this differential equation. Show all work.

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11. A spring is arranged horizontally. The mass at the end of the spring is $m = 1$ kilogram. The damping constant is $\beta = 4$ kilograms/sec. The spring constant is $k = 4$ kilograms/sec^2. There is an additional force of $f(t) = 6t^2e^{-2t}$ acting on the mass. Let $y(t)$ be the position of the mass relative to its equilibrium position after t seconds and it satisfies the initial conditions $y(0) = y'(0) = 0$. Use the method of Laplace transforms to find the formula for $y(t)$. Show all work.

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12. Solve the following differential equation:

$$\frac{d^2y}{dt^2} + y = 2t\cdot\mathcal{U}\left(t-\frac{\pi}{2}\right) \qquad \text{where } y(0)=0,\ y'(0)=1$$