MA 345 Practice Final Exam

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1. Let $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$, find the general solution of the matrix equation $\frac{d\vec{\mathbf{X}}}{dt} = \mathbf{A}\vec{\mathbf{X}}$.

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2. If we substitute $y = \sum a_n x^n$ into the differential equation $\frac{d^2y}{dx^2} - 2y = 0$, we will get a formula relating a_{n+2} to a_n (the recurrence relation). Find this formula.

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3. Find the general solution $y = y(t)$ of each of the following differential equations:

a) $\dfrac{d^2y}{dt^2} - 5\dfrac{dy}{dt} + 6y = 0$

b) $\dfrac{d^2y}{dt^2} - 6\dfrac{dy}{dt} + 9y = 0$

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4. Use method of undetermined coefficients to solve the following nonhomogeneous equation:

$$\frac{d^2y}{dt^2} - 4\frac{dy}{dt} + 3y = e^{3t}$$

5. Solve using Laplace transforms:

$$\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 9y = e^{3t}\mathcal{U}(t-1) \qquad \text{where } y(0) = 0 \text{ and } y'(0) = 0$$

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6. Solve the differential equation:

$$(2xy + \sqrt{y})\ dx + \frac{1}{2}x^2\ dy = 0 \quad \text{where } y(1) = 1$$

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7. One of the following numbers is an eigenvalue of $\begin{pmatrix} 0 & 1 \\ -2 & 3 \end{pmatrix}$.
Which one?

a) -2 **b)** -1 **c)** 0 **d)** 2 **e)** 3

8. One of the following vectors is an eigenvector of $\begin{pmatrix} 0 & 1 \\ -2 & 3 \end{pmatrix}$. Which one?

a) $\binom{1}{2}$ **b)** $\binom{2}{1}$ **c)** $\binom{1}{0}$ **d)** $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ **e)** $\binom{0}{1}$

9. Let $\mathcal{U}(t)$ denote the unit step function. Find the Laplace transform of $e^t\mathcal{U}(t-2)$

a) $\frac{e^{1-2s}}{s-2}$ **b)** $\frac{e^{2-2s}}{s-1}$ **c)** $\frac{e^{-s}}{s-2}$ **d)** $\frac{e^{-s}}{s-1}$ **e)** $\frac{e^{2-s}}{s-1}$

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10. Find the inverse Laplace transform of $\frac{s}{(s+1)(s+2)}$

a) $e^{-2t} + 2e^{-t}$ **b)** $e^{-t} - e^{-2t}$ **c)** $2e^{-2t} - e^{-t}$

d) $e^{t} + 2e^{2t}$ **e)** $e^{t} + e^{2t}$

11. Which of the following would be an integrating factor μ for the differential equation:

$$y^2 dx + (e^x - 2y)\ dy = 0$$

a) $\frac{1}{y^2}$ **b)** e^x **c)** e^y **d)** e^{-x} **e)** e^{-y}

12. Let $y = y(x)$ be the solution of the equation $\frac{dy}{dx} = \frac{y}{x} + \frac{x}{y}$. If we make the substitution $v = \frac{y}{x}$, which of the following will be the resulting equation that determines $v = v(x)$?

a) $\frac{dv}{dx} = v + \frac{1}{v}$ **b)** $\frac{dv}{dx} = \frac{v}{x}$ **c)** $\frac{dv}{dx} = \frac{x}{v}$

d) $\frac{dv}{dx} = xv$ **e)** $\frac{dv}{dx} = \frac{1}{xv}$

13. If the *Method of Undetermined Coefficients* was used to solve the differential equation $\frac{d^2y}{dx^2} - y = e^{2x} + x^2$, which of the following would be the general form of the particular solution y_p ?

a) $ae^{2x} + bx^2$

b) $ae^{2x} + b_1 + b_2x + b_3x^2$

c) ax^2e^{2x}

d) $ae^x + be^{-x}$

e) $ae^x + be^{-x} + c_1e^{2x} + c_2x^2$

14. The general form of the particular solution of $y'' - y' = x$ is $y_p = ax + bx^2$.
Find the coefficient b

a) $-\frac{1}{2}$ **b)** -1 **c)** 0 **d)** 1 **e)** $\frac{1}{2}$

15. According to the Exponential Shift Theorem, the expression $(D+1)^4\,(e^{-x}\sin x)$ is equal to:

a) $e^{-x}\sin x$

b) $-e^{-x}\cos x$

c) $e^{-x}(\sin x+\cos x)$

d) $e^{-x}(\sin x-\cos x)$

e) $e^{-x}+\sin x$

16. The expression $y = c_1e^{-x}\sin x + c_2e^{-x}\cos x$ is the general solution for one of the following differential equations. Which one?

a) $(D^2 - 1)y = 0$

b) $(D^2 + 2D + 2)y = 0$

c) $(D^2 - 2D + 1)y = 0$

d) $(D^2 - 2D + 2)y = 0$

e) $(D^2 + 2D + 1)y = 0$