MA 345 **Practice Exam 2** **Dr. E. Jacobs**

Part 1 - Multiple Choice Section

1. Consider the following system of linear equations:

$$\begin{aligned} x + y + z &= 0 \\ 2x + 3y + 2z &= 0 \\ -x - y \quad &= 1 \end{aligned}$$

If we represent this system of equations as an augmented matrix and then apply the matrix reduction operations, which of the following matrices will be the result?

a) $\left(\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right)$ **b)** $\left(\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right)$ **c)** $\left(\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$

d) $\left(\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right)$ **e)** $\left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right)$

---

2. For the matrices given as choices in problem 1, which represents a system of equations that has an infinite number of solutions?

a) $\left(\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right)$ **b)** $\left(\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right)$ **c)** $\left(\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$

d) $\left(\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right)$ **e)** $\left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right)$

---

3. For the matrices given as choices in problem 1, which represents a system of equations that has no solutions at all?

a) $\left(\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right)$ **b)** $\left(\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right)$ **c)** $\left(\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$

d) $\left(\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right)$ **e)** $\left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right)$

---

4. Let $\mathbf{A}$ be defined as follows:

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

Which of the following statements is true about $\mathbf{A}^{-1}$ (the inverse of matrix $\mathbf{A}$) ?

a) $\mathbf{A}^{-1} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ **b)** $\mathbf{A}^{-1} = \begin{pmatrix} -1 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & -1 \end{pmatrix}$ **c)** $\mathbf{A}^{-1} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$

d) $\mathbf{A}^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ **e)** $\mathbf{A}^{-1}$ doesn't exist

5. There are two distinct eigenvalues for $\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 0 & 4 \end{pmatrix}$.
One of them is $\lambda = 4$. What's the other eigenvalue?

a) 0 **b)** 1 **c)** -1 **d)** 4 **e)** -4

6. Once again, let $\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 0 & 4 \end{pmatrix}$. What is the eigenvector that corresponds to eigenvalue $\lambda = 4$?

a) $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ **b)** $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$ **c)** $\begin{pmatrix} 1 \\ 4 \end{pmatrix}$ **d)** $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ **e)** $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

7. What is the general solution of the differential equation $\big(D^2 - 9\big)\, y = 0$

a) $y = c_1 + c_2 e^{3x}$ **b)** $y = c_1 e^{3x} + c_2 e^{-3x}$ **c)** $y = c_1 e^{x} + c_2 e^{3x}$
d) $y = c_1 e^{-3x} + c_2 x e^{-3x}$ **e)** $y = c_1 \cos(3x) + c_2 \sin(3x)$ **f)** $y = c_1 e^{3x} \cos x + c_2 e^{3x} \sin x$

8. What is the general solution of the differential equation $\big(D^2 - 3D\big)\, y = 0$

a) $y = c_1 + c_2 e^{3x}$ **b)** $y = c_1 e^{3x} + c_2 e^{-3x}$ **c)** $y = c_1 e^{x} + c_2 e^{3x}$
d) $y = c_1 e^{-3x} + c_2 x e^{-3x}$ **e)** $y = c_1 \cos(3x) + c_2 \sin(3x)$ **f)** $y = c_1 e^{3x} \cos x + c_2 e^{3x} \sin x$

9. What is the general solution of the differential equation $\big(D^2 + 9\big)\, y = 0$

a) $y = c_1 + c_2 e^{3x}$ **b)** $y = c_1 e^{3x} + c_2 e^{-3x}$ **c)** $y = c_1 e^{x} + c_2 e^{3x}$
d) $y = c_1 e^{-3x} + c_2 x e^{-3x}$ **e)** $y = c_1 \cos(3x) + c_2 \sin(3x)$ **f)** $y = c_1 e^{3x} \cos x + c_2 e^{3x} \sin x$

10. What is the general solution of the differential equation $\big(D^2 + 6D + 9\big)\, y = 0$

a) $y = c_1 + c_2 e^{3x}$ **b)** $y = c_1 e^{3x} + c_2 e^{-3x}$ **c)** $y = c_1 e^{x} + c_2 e^{3x}$
d) $y = c_1 e^{-3x} + c_2 x e^{-3x}$ **e)** $y = c_1 \cos(3x) + c_2 \sin(3x)$ **f)** $y = c_1 e^{3x} \cos x + c_2 e^{3x} \sin x$

11. What is the general solution of the differential equation $\big(D^2 - 4D + 3\big)\, y = 0$

a) $y = c_1 + c_2 e^{3x}$ **b)** $y = c_1 e^{3x} + c_2 e^{-3x}$ **c)** $y = c_1 e^{x} + c_2 e^{3x}$
d) $y = c_1 e^{-3x} + c_2 x e^{-3x}$ **e)** $y = c_1 \cos(3x) + c_2 \sin(3x)$ **f)** $y = c_1 e^{3x} \cos x + c_2 e^{3x} \sin x$

12. What is the general solution of the differential equation $\big(D^2 - 6D + 10\big)\, y = 0$

a) $y = c_1 + c_2 e^{3x}$ **b)** $y = c_1 e^{3x} + c_2 e^{-3x}$ **c)** $y = c_1 e^{x} + c_2 e^{3x}$
d) $y = c_1 e^{-3x} + c_2 x e^{-3x}$ **e)** $y = c_1 \cos(3x) + c_2 \sin(3x)$ **f)** $y = c_1 e^{3x} \cos x + c_2 e^{3x} \sin x$

13. The complex expression $e^{\alpha + i\beta t} + e^{\alpha - i\beta t}$ is equal to one of the following expressions. Which one?

a) e^{α} **b)** $2e^{\alpha} \sin(\beta t)$ **c)** $2e^{\alpha} \cos(\beta t)$
d) $\cos(\beta t) + \sin(\beta t)$ **e)** $e^{\alpha}(\cos(\beta t) + i \sin(\beta t))$

Part 2 - Free Response Section

14. Define matrices $\mathbf{A}$, $\mathbf{P}$, $\mathbf{Q}$, $\vec{\mathbf{X}}$ and $\vec{\mathbf{0}}$ to be:

$$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 0 & 4 \end{pmatrix} \qquad \mathbf{P} = \begin{pmatrix} 1 & 1 \\ 0 & 4 \end{pmatrix} \qquad \mathbf{Q} = \begin{pmatrix} 4 & -1 \\ 0 & 1 \end{pmatrix} \qquad \vec{\mathbf{X}} = \begin{pmatrix} x \\ y \end{pmatrix} \qquad \vec{\mathbf{0}} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Calculate each of the following matrix expressions:

a) $\mathbf{P}^2$ - 5**P** **b)** **QAP**

Solve each of the following equations for $\vec{\mathbf{X}}$:

c) $\mathbf{A}\vec{\mathbf{X}} = \vec{\mathbf{0}}$ **d)** $\mathbf{A}\vec{\mathbf{X}} = \vec{\mathbf{X}}$

---

15. Let $\vec{\mathbf{X}} = \begin{pmatrix} x \\ y \end{pmatrix}$ where x and y depend on a variable t. Solve the following matrix differential equation:

$$\frac{d\vec{\mathbf{X}}}{dt} = \begin{pmatrix} 0 & 1 \\ -8 & -6 \end{pmatrix} \vec{\mathbf{X}} \qquad \text{where } \vec{\mathbf{X}}(0) = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

---

16. Let $y(t)$ be the displacement of a mass at the end of a horizontal spring. If the mass is $m = 1$ kg, and the damping constant is $\beta = 2$ kg/sec and the spring constant is $k = 3$ kg/sec^2, then $y(t)$ can be found by solving the following differential equation.

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 3y = 0$$

Solve this differential equation and determine all coefficients in your answer so that $y(t)$ satisfies the initial conditions $y(0) = 0$ and $y'(0) = 2$ Show all work.