Differential Equations and Matrix Methods
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Today's Topic : The Distinct Root Case

nth Order Linear Homogeneous Equation

$$a_n \frac{d^n y}{dx^n} + a_{n-1} \frac{d^{n-1} y}{dx^{n-1}} + \cdots + a_1 \frac{dy}{dx} + a_0 y = 0$$

$$\frac{d^2y}{dt^2} + y = 0$$

$$\frac{d^2y}{dt^2} + \frac{dy}{dt} = 0$$

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = 0$$

$$\frac{d^3y}{dt^3} - \frac{dy}{dt} = 0$$

Homogeneous Equations

$$\frac{d^2y}{dt^2} + y = \sin t$$

$$\frac{d^2y}{dt^2} + \frac{dy}{dt} = t^2$$

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = t + \cos t$$

$$\frac{d^3y}{dt^3} - \frac{dy}{dt} = e^{2t}$$

Nonhomogeneous Equations

The general solution of $\frac{d^3y}{dt^3} - \frac{dy}{dt} = 0$ is:

$$y = c_1 e^{0t} + c_2 e^{t} + c_3 e^{-t}$$

Every solution of $y''' - y' = 0$ is a linear combination of $y_1 = e^{0t}$, $y_2 = e^{t}$ and $y_3 = e^{-t}$

The functions y_1, y_2 and y_3 are linearly independent

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The functions y_1, y_2 and y_3 are linearly independent
The set of functions $\{y_1,\ y_2,\ y_3\}$ is a *basis* for our solution space.

$$\frac{d^2y}{dt^2} - 4y = 0$$

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$$y = c_1 e^{2t} + c_2 e^{-2t}$$

If $y(0) = 0$ then $0 = c_1 + c_2$ so $c_2 = -c_1$

$$y = c_1 e^{2t} - c_1 e^{-2t}$$

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$$y' = 2c_1 e^{2t} + 2c_1 e^{-2t}$$

$2 = y'(0)$ implies that $2 = 2c_1 + 2c_1$

$$y = \frac{1}{2} e^{2t} - \frac{1}{2} e^{-2t}$$

The Hyperbolic Functions

$$\sinh x = \frac{1}{2}\left(e^x - e^{-x}\right)$$

$$\cosh x = \frac{1}{2}\left(e^x + e^{-x}\right)$$

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$2 = y'(0)$ implies that $2 = 2c_1 + 2c_1$

$$y = \frac{1}{2}e^{2t} - \frac{1}{2}e^{-2t} = \sinh 2t$$

Find $y = y(x)$ that solve:

$$\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = 0$$

Start by looking for solutions of the form e^{rx}

Find $y = y(x)$ that solve:

$$\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = 0$$

Start by looking for solutions of the form e^{rx}

$$r^3e^{rx} - 2r^2e^{rx} - 2re^{rx} = 0$$

$$r^3 - 2r^2 - 2r = 0$$

Find $y = y(x)$ that solve:

$$\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = 0$$

$$\left(D^3 - 2D^2 - 2D\right) y = 0$$

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$$r^3 - 2r^2 - 2r = 0$$

$$r\left(r^2 - 2r - 2\right) = 0$$

$r = 0$ is one solution.
For the others, solve $r^2 - 2r - 2 = 0$ to get $r = 1 \pm \sqrt{3}$

$$y = c_1 e^{0x} + c_2 e^{(1+\sqrt{3})x} + c_3 e^{(1-\sqrt{3})x}$$

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$$\left(D^4 - 10D^2 + 9\right) y = 0$$

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$$\left(D^2 - 1\right)\left(D^2 - 9\right) y = 0$$

Find $y = y(x)$ that solves:

$$\left(D^4 - 10D^2 + 9\right) y = 0$$

$$(D-1)(D+1)(D-3)(D+3)y = 0$$

If we substitute e^{rx} we would get:

$$(r-1)(r+1)(r-3)(r+3) = 0$$

$$r = 1 \qquad r = -1 \qquad r = 3 \qquad r = -3$$

Find $y = y(x)$ that solves:

$$\left(D^4 - 10D^2 + 9\right) y = 0$$

If we substitute e^{rx} we would get:

$$(r-1)(r+1)(r-3)(r+3) = 0$$

$$r = 1 \qquad r = -1 \qquad r = 3 \qquad r = -3$$

$$y = c_1 e^{1x} + c_2 e^{-x} + c_3 e^{3x} + c_4 e^{-3x}$$

Solve:

$$\left(D^2 - 4D + 4\right) y = 0$$

Substitute e^{rx}

$$r^2 - 4r + 4 = 0$$

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$$\left(D^2 - 4D + 4\right) y = 0$$

Substitute e^{rx}

$$r^2 - 4r + 4 = 0$$

$$(r-2)(r-2) = 0$$

$$r = 2$$

$$y = c_1 e^{2x} + c_2(\mathbf{???})$$