MA 441 *Advanced Engineering Mathematics I* Dr. E. Jacobs

Assignments - Spring 2014

The main texts for this course are "Calculus" by James Stewart and "Fundamentals of Differential Equations by Nagle, Saff and Snider. However, you may use "Advanced Engineering Mathematics" by Zill and Wright if you prefer. The chapters and problems referred to in this assignment sheet are from the Stewart book and the Nagle/Saff/Snider book. References to the appropriate chapters and problems in the Zill book are available on request.

Assignment 1. *Vector Fields and Flow Lines*

Read Section 16.1
You should be able to do the following problems:
Section 16.1/Problem 1 - 36, 35 - 36
Hand in the following problems:

1.. Plot each of the following vector fields by drawing arrows of approximately the correct length and direction at the points: (0, 2), (0, 4), (2, 2), (2, 4), (4, 2)

a. $\vec{\mathbf{F}} = \left\langle \frac{y}{2}\,,\ -\frac{x}{2} \right\rangle$ **b.** $\vec{\mathbf{F}} = \left\langle \frac{-x}{\sqrt{x^2+y^2}}\,,\ \frac{-y}{\sqrt{x^2+y^2}} \right\rangle$

Problems 2 - 3. For each of the vector fields given in problem 1, find the general expressions for the flow lines in two dimensions.

Problems 4 - 5. For each of the following vector fields, find the expressions for the flow lines in three dimensions that pass through the given point $\vec{\mathbf{r}}_0$. Express your answers in the form $\vec{\mathbf{r}} = \langle x(t),\ y(t),\ z(t)\rangle$

4. $\vec{\mathbf{F}} = \left\langle 2xz,\ 2xz,\ z^2+4 \right\rangle \quad \vec{\mathbf{r}}_0 = \langle 4,1,0\rangle$ **5.** $\vec{\mathbf{F}} = \langle x,\ x-y,\ z\rangle \quad \vec{\mathbf{r}}_0 = \langle 1,\ 1/2,\ 2\rangle$

Assignment 2. *Parametrically Defined Surfaces and Surface Area*

Read Section 16.6
You should be able to do the following problems:
Section 16.6/Problem 19 - 20, 23 - 25, 39 - 50
Hand in the following problems:

Problems 1 - 3. Each of the surfaces in problems 1, 2 and 3 are defined parametrically. Draw each surface on an xyz axis.

1 $\langle x,\ y,\ z\rangle = \langle u,\ v,\ 1-u-v\rangle$ where $0 \le u \le 1$ and $0 \le v \le 1$

2. $\langle x,\ y,\ z\rangle = \langle 2\cos u,\ v,\ 2\sin u\rangle$ where $0 \le u \le \pi$ and $0 \le v \le 2$

3. $\langle x,\ y,\ z\rangle = \langle v\cos u,\ v\sin u,\ 1-v\rangle$ where $0 \le u \le 2\pi$ and $0 \le v \le 1$

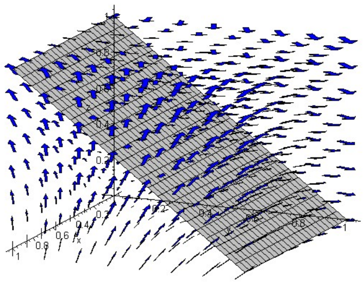
4. If a surface Ω is described parametrically by the equation $\vec{\mathbf{r}} = \langle x(u,v),\ y(u,v), z(u,v)\rangle$ then the area of the surface can be found using the formula $\text{Area}(\Omega) = \iint \left|\frac{\partial\vec{\mathbf{r}}}{\partial u} \times \frac{\partial\vec{\mathbf{r}}}{\partial v}\right|\, du\, dv$. Use this formula to find the area of the surface in problem 2.

5. Find the surface area of the surface in problem 3.

Assignment 3. *Surface Integrals*

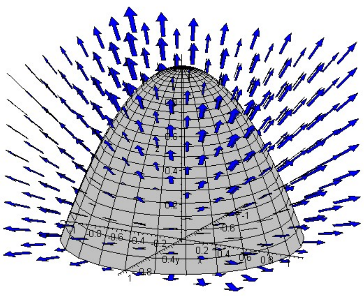
Read Section 16.7
You should be able to do the following problems:
Section 16.7/Problem 21 - 32
Hand in the following problems:

1. Let S_P be the planar surface described by the equation $\vec{\mathbf{r}} = \langle x,\ y,\ 1-y\rangle$ for $0 \le x \le 1$ and $0 \le y \le 1$. Let $\vec{\mathbf{F}} = \langle z,\ y,\ x\rangle$.



Calculate the surface integral $\iint_{S_P} \vec{\mathbf{F}} \bullet \vec{\mathbf{n}}\, dS$.

2. Let S be the parabolic surface described by the equation $\vec{\mathbf{R}} = \langle r\cos\theta,\ r\sin\theta,\ 1-r^2\rangle$ for $0 \le r \le 1$ and $0 \le \theta \le 2\pi$. Let $\vec{\mathbf{F}} = \langle 2x,\ 2y,\ 2z\rangle$.



Calculate the surface integral $\iint_S \vec{\mathbf{F}} \bullet \vec{\mathbf{n}}\, dS$.

3. Let S be the sphere described by the equation $\vec{\mathbf{r}} = \langle \cos\theta\sin\phi,\ \sin\theta\sin\phi,\ \cos\phi\rangle$ for $0 \le \theta \le 2\pi$ and $0 \le \phi \le \pi$. Let $\vec{\mathbf{F}} = \langle -y,\ x,\ 0\rangle$. Calculate the surface integral $\iint_S \vec{\mathbf{F}} \bullet \vec{\mathbf{n}}\, dS$.

4. Let Ω be the surface described in Assignment 2 Problem 2. Let $\vec{\mathbf{F}} = \langle x,\ y,\ y+z\rangle$ Calculate the surface integral $\iint_\Omega \vec{\mathbf{F}} \bullet \vec{\mathbf{n}}\, dS$.

5. Again, let Ω be the surface described in Assignment 2 Problem 2 and let $\vec{\mathbf{F}} = \langle x,\ y,\ y+z\rangle$. Let V be the solid bounded from above by Ω and below by the xy plane for $0 \le y \le 2$. Let S be the *closed surface* that completely surrounds V. S includes not only Ω, but also two semicircular sides at $y=0$ and $y=2$ and a rectangular base on the xy plane. Calculate the surface integral $\iint_S \vec{\mathbf{F}} \bullet \vec{\mathbf{n}}\, dS$.

Assignment 4. *Divergence and Curl*

Read Section 16.5
You should be able to do the following problems:
Section 16.5/Problem 1 - 18, 23 - 32
Hand in the following problems:

1. Find div $\vec{\mathbf{F}}$, given that:
(a) $\vec{\mathbf{F}} = e^{xy}\,\vec{\mathbf{i}} + \sin xy\,\vec{\mathbf{j}} + \cos^2\ zx\,\vec{\mathbf{k}}$.
(b) $\vec{\mathbf{F}} = \nabla\phi$, where $\phi = 3x^2y^3z$.

2. Find **curl** $\vec{\mathbf{F}}$ if $\vec{\mathbf{F}} = z^2x\vec{\mathbf{i}} + y^2z\vec{\mathbf{j}} - z^2y\vec{\mathbf{k}}$.

3. Given $\vec{\mathbf{F}}(x, y, z) = x^2y\vec{\mathbf{i}} + z\vec{\mathbf{j}} - (x + y - z)\vec{\mathbf{k}}$, find
(a) $\nabla\bullet\vec{\mathbf{F}}$ (b) $\nabla\times\vec{\mathbf{F}}$ (c) $\nabla(\nabla\bullet\vec{\mathbf{F}})$

4. Let $\phi = \phi(x, y, z)$ be some scalar-valued function. Let $\vec{\mathbf{F}} = \langle F_1,\ F_2,\ F_3\rangle$ be some vector field where each coordinate may depend on x, y and z. Classify each of the following expressions as *vector*, *scalar* or *meaningless*.

$$\nabla\bullet(\nabla\phi) \qquad \nabla\times(\nabla\bullet\vec{\mathbf{F}}) \qquad \nabla\bullet(\nabla^2\phi) \qquad \vec{\mathbf{F}}\bullet\nabla\phi \qquad \vec{\mathbf{F}}\times\nabla\phi$$

5. Let $\phi(x, y) = x^2y^4 + xy$. Let $\vec{\mathbf{F}} = \langle x,\ -z,\ y\rangle$ Calculate each of the following:

a. $\nabla\phi$ **b.** $\nabla^2\phi$ **c.** $\nabla\bullet\vec{\mathbf{F}}$ **d.** $\nabla\times\vec{\mathbf{F}}$

6. Let $\phi(x, y) = \frac{1}{2}y^2\ln x$ and $\vec{\mathbf{F}} = \left(x^2y\right)\vec{\mathbf{i}} + \left(x + y^2\right)\vec{\mathbf{j}} + \vec{\mathbf{k}}$. Calculate each of the following:

a. $\nabla\bullet\vec{\mathbf{F}}$ **b.** $\nabla\times\vec{\mathbf{F}}$ **c.** $\nabla^2\phi$ **d.** $\nabla\times\nabla\phi$

7. Let $\vec{\mathbf{F}} = \left(4 + \frac{y}{\sqrt{x}}\right)\vec{\mathbf{i}} + (2\sqrt{x} + 1)\,\vec{\mathbf{j}}$
a. Verify that **curl** $\vec{\mathbf{F}} = \vec{\mathbf{0}}$
b. Find a scalar function $\phi(x, y)$ such that $\vec{\mathbf{F}} = \nabla\phi$

8. Let f and g be any differentiable scalar-valued functions of x and y. Prove the following identity:

$$\nabla\bullet(f\nabla g) = f\nabla^2 g + \nabla f\bullet\nabla g$$

9. Let $\vec{\mathbf{r}} = \langle x,\ y,\ z\rangle$. Calculate div$(\vec{\mathbf{r}})$ and **curl**$(\vec{\mathbf{r}})$.

10. If $\vec{\mathbf{r}} = \langle x,\ y,\ z\rangle$ then $|\vec{\mathbf{r}}| = \left(x^2 + y^2 + z^2\right)^{\frac{1}{2}}$. and $|\vec{\mathbf{r}}|^n = \left(x^2 + y^2 + z^2\right)^{\frac{n}{2}}$, where n is any constant. Find a general formula for $\nabla\left(|\vec{\mathbf{r}}|^n\right)$. Write your answer in simplest form.

Assignment 5. *Line Integrals*

Read Section 16.2, 16.3

You should be able to do the following problems:

Section 16.2/Problem 1 - 30, Section 16.3/Problem 3 - 18

Hand in the following problems:

1. The following equation describes a parabolic path C:

$$\vec{\mathbf{r}}(t) = \langle t,\ 2,\ 1 - t^2 \rangle \qquad -1 \leq t \leq 1$$

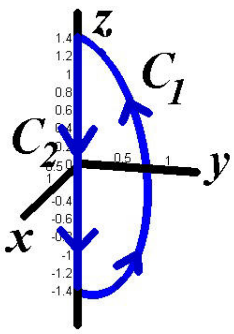
If $\vec{\mathbf{F}} = \langle 3z,\ x + y + z,\ 2x \rangle$, calculate the line integral $\int_C \vec{\mathbf{F}} \bullet d\vec{\mathbf{r}}$

2. The following equation describes a semicircle C_1:

$$\vec{\mathbf{r}}(t) = \langle \cos t,\ \cos t,\ \sqrt{2}\sin t \rangle \qquad -\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$$

If $\vec{\mathbf{F}} = \langle -z,\ -x,\ \pi + x \rangle$, calculate the line integral $\int_{C_1} \vec{\mathbf{F}} \bullet d\vec{\mathbf{r}}$

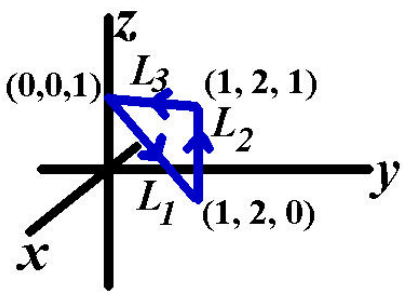
3. Let C_1 be the same curve defined in problem 2 and again let $\vec{\mathbf{F}} = \langle -z,\ -x,\ \pi + x \rangle$. Let C_2 be the straight line segment from $(0,\ 0,\ \sqrt{2})$ to $(0,\ 0,\ -\sqrt{2})$. Together, path C_1 followed by path C_2 form a closed loop C.



Calculate $\oint_C \vec{\mathbf{F}} \bullet d\vec{\mathbf{r}}$ by combining $\int_{C_1} \vec{\mathbf{F}} \bullet d\vec{\mathbf{r}}$ with $\int_{C_2} \vec{\mathbf{F}} \bullet d\vec{\mathbf{r}}$.

4. Let L_1 be the straight line segment from $(0,\ 0,\ 1)$ to $(1,\ 2,\ 0)$ and let $\vec{\mathbf{F}} = \langle z + 2x,\ 2y,\ x \rangle$. Calculate the line integral $\int_{L_1} \vec{\mathbf{F}} \bullet d\vec{\mathbf{r}}$

5. Let L_1 and $\vec{\mathbf{F}}$ be the same as defined in problem 4. Let L_2 be the straight line segment from $(1,\ 2,\ 0)$ to $(1,\ 2,\ 1)$. Let L_3 be the straight line segment from $(1,\ 2,\ 1)$ to $(0,\ 0,\ 1)$. Path L_1 followed by path L_2 and then by path L_3 forms a closed triangular loop C. Calculate $\oint_C \vec{\mathbf{F}} \bullet d\vec{\mathbf{r}}$ by combining $\int_{L_1} \vec{\mathbf{F}} \bullet d\vec{\mathbf{r}}$, $\int_{L_2} \vec{\mathbf{F}} \bullet d\vec{\mathbf{r}}$ and $\int_{L_3} \vec{\mathbf{F}} \bullet d\vec{\mathbf{r}}$.



Assignment 6. *Stokes' Theorem*

Read Section 16.4, 16.8

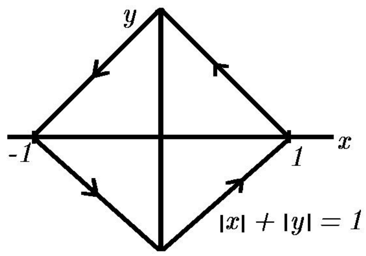
You should be able to do the following problems:

Section 16.4/Problem 11 - 24, Section 16.8/Problem 1 - 15

Hand in the following problems:

1. Let C be the circle of radius 1 around the origin in the yz plane. Let $\vec{\mathbf{F}} = \langle 0,\ y+4z,\ 6y+z \rangle$. Use Stokes' Theorem to calculate $\oint_C \vec{\mathbf{F}} \bullet d\vec{\mathbf{r}}$.

2. Let C be the square in the xy plane with equation $|x| + |y| = 1$.



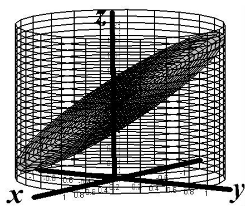
Let $\vec{\mathbf{F}} = -y\vec{\mathbf{i}} + x\vec{\mathbf{j}}$. Calculate $\oint_C \vec{\mathbf{F}} \bullet d\vec{\mathbf{r}}$ in two ways:

a) Add up the integrals over all four line segments of C.

b) Use Stokes' Theorem.

3. Let $\vec{\mathbf{F}} = -2y\vec{\mathbf{i}} + x\vec{\mathbf{j}} + x\vec{\mathbf{k}}$

Let C be the intersection of the plane $z = 1 + y$ with the cylinder $x^2 + y^2 = 1$ and orient C counterclockwise as viewed from above.



Use Stokes' Theorem to calculate the line integral $\oint_C \vec{\mathbf{F}} \bullet d\vec{\mathbf{r}}$.

4. Use Stokes' Theorem to calculate the closed loop line integral in Assignment 5 Problem 3.

5. Use Stokes' Theorem to calculate the closed loop line integral in Assignment 5 Problem 5.

Assignment 7. *Stokes' Theorem and Green's Theorem*

Read Section 16.4, 16.8
You should be able to do the following problems:
Section 16.4/Problem 11 - 24, Section 16.8/Problem 1 - 15
Hand in the following problems:

1. Let R be the region in the first and second quadrants of the xy plane bounded by the lines $y = x$, $y = -x$ and the circle $x^2 + y^2 = 4$. Let C be the boundary of this region, traversed in a counterclockwise direction. Let $\vec{\mathbf{v}} = x^2 y \vec{\mathbf{i}} + xy^2 \vec{\mathbf{j}}$. Use Green's Theorem to evaluate $\oint_C \vec{\mathbf{v}} \bullet d\vec{\mathbf{r}}$

2. Let C denote the triangular path connecting $(0, 0, 0)$ to $(1, 0, 0)$ to $(0, 1, 0)$ and back to $(0, 0, 0)$. C forms the boundary of a triangular region S_{xy} in the xy plane. Show how Green's Theorem can be used to calculate:

$$\oint_C xy\,dx + \frac{1}{2}\left(x^2 + x + y\right)\,dy$$

3. Let R be the region in the yz plane below the curve $z = \sqrt{y}$ and above the y-axis between $y = 0$ and $y = 2$. Let C be the curve surrounding this region R. Assume that C is traversed in the counterclockwise direction as viewed from the positive x-axis.

Let $\vec{\mathbf{F}} = \frac{1}{2} z^2 \vec{\mathbf{j}} + y \vec{\mathbf{k}}$. Use Stokes' Theorem to evaluate $\oint_C \vec{\mathbf{F}} \bullet d\vec{\mathbf{r}}$

4. Let S be the portion of the surface $z = 1 + y^2$ bounded by the vertical planes $x = 0$, $x = 1$, $y = 0$ and $y = 1$. Let C be the curve that forms the boundary of S. You may assume that C is traversed in the counterclockwise direction. Define $\vec{\mathbf{F}}$ as follows:

$$\vec{\mathbf{F}} = z\vec{\mathbf{i}} + 2x\vec{\mathbf{j}} + 3y\vec{\mathbf{k}}$$

Evaluate $\oint_C \vec{\mathbf{F}} \bullet d\vec{\mathbf{r}}$ using Stokes' Theorem.

5. Let H be the hemisphere described by the equation

$$z = \sqrt{1 - x^2 - y^2}$$

Let $\vec{\mathbf{F}} = z\vec{\mathbf{i}}$. Use any legitimate method to calculate the surface integral:

$$\int\!\!\int_H (\nabla \times \vec{\mathbf{F}}) \bullet \vec{\mathbf{n}}\,dS$$

Assignment 8. *Divergence Theorem*

Read Section 16.9

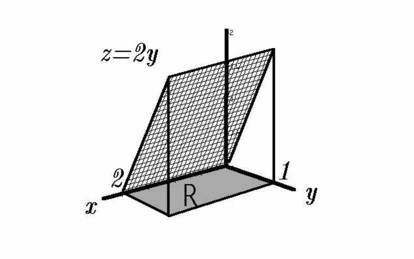
You should be able to do the following problems:

Section 16.9/Problem 1 - 15

Hand in the following problems:

1. Let T be the three dimensional solid bounded from above by the paraboloid $z = 1 - x^2 - y^2$ and from below by $z = 0$. Let S be the closed surface that completely surrounds T. Let $\vec{\mathbf{F}} = \langle 2x,\ 2y,\ 2z\rangle$. Use the Divergence Theorem to calculate $\iint_S \vec{\mathbf{F}} \bullet \vec{\mathbf{n}}\, dS$. Compare your answer to Problem 2 on Assignment 3.

2. Let R be the rectangular region with $0 \le x \le 2$ and $0 \le y \le 1$. Let V be the three dimensional region that lies above R but below the plane $z = 2y$. Let S be the closed surface surrounding V.



Let $\vec{\mathbf{F}} = \langle x + y,\ y + z,\ z - x\rangle$. Use the Divergence Theorem to evaluate the surface integral $\iint_S \vec{\mathbf{F}} \bullet \vec{\mathbf{n}}\ dS$.

3. Let T be the three dimensional solid bounded from above by the half cylinder $z = \sqrt{1 - x^2}$ and from below by $z = 0$ for $0 \le y \le 2$. Let S be the closed surface that completely surrounds T. Let $\vec{\mathbf{F}} = \langle x,\ y,\ y + z\rangle$. Use the Divergence Theorem to calculate $\iint_S \vec{\mathbf{F}} \bullet \vec{\mathbf{n}}\, dS$. Compare your answer to Problem 5 on Assignment 3.

4. Let T be the quarter sphere solid that is inside $x^2 + y^2 + z^2 = 1$ for $x \ge 0$ and $y \ge 0$. Let S be the closed surface that completely surrounds T. Let $\vec{\mathbf{F}} = (3z + y)\vec{\mathbf{k}}$. Use the Divergence Theorem to calculate $\iint_S \vec{\mathbf{F}} \bullet \vec{\mathbf{n}}\, dS$

5. Let $\vec{\mathbf{F}} = \left(x^2 + y^2\right)\vec{\mathbf{j}}$. Let S be the closed surface surrounding the rectangular box with vertices:

$(0,0,0)$	$(1,0,0)$	$(1,2,0)$	$(0,2,0)$
$(0,0,3)$	$(1,0,3)$	$(1,2,3)$	$(0,2,3)$

Evaluate $\iint_S \vec{\mathbf{F}} \bullet \vec{\mathbf{n}}\, dS$ using the Divergence Theorem

Assignment 9. *Fourier Series*

Read Section 10.3, 10.4 (Nagle, Saff and Snider)
You should be able to do the following problems:
Section 10.3/Problem 1 - 15, Section 10.4/Problem 1 - 16
Hand in the following problems:

A Fourier series is an expression of the form:

$$a_0 + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{L} + b_n \sin \frac{n\pi x}{L} \right)$$

In problems 1 - 4, you are given four different functions. In each case, calculate the Fourier coefficient a_0 as well as general formulas for a_n and b_n when n is a positive integer. Use $L = \pi$ when you do this. Simplify your answers as much as possible. It helps to remember that $\sin n\pi = 0$ and $\cos n\pi = (-1)^n$. You are advised to pay attention to whether or not the function is even or odd - this will speed your calculations. Also, for problems 1 - 4, plot the following function for $-\pi \le x \le \pi$:

$$G(x) = a_0 + \sum_{n=1}^{3} (a_n \cos nx + b_n \sin nx)$$

and staple the printed plots to your assignment. I like using **Maple** for this task, but you may use **MatLab** or any other appropriate software that you are comfortable with to do this.

1.

$$f(x) = \begin{cases} \pi - x & \text{for } x \ge 0 \\ -\pi - x & \text{for } x < 0 \end{cases}$$

2.

$$g(x) = \begin{cases} \pi & \text{for } -\frac{\pi}{2} \le x \le \frac{\pi}{2} \\ 0 & \text{for all other } x \text{ values} \end{cases}$$

3.

$$\phi(x) = \pi^2 - x^2$$

4.

$$\psi(x) = \sinh x$$

5. For this next problem, take $L = 1$ so the Fourier series will have the form $a_0 + \sum(a_n \cos n\pi x + b_n \sin n\pi x)$. Find a_n and b_n and express the Fourier series in summation notation for the following odd function $U(x)$. You may omit the plot of $G(x)$ for problem 5.

$$U(x) = \begin{cases} x - x^2 & \text{for } \quad 0 \le x \le 1 \\ x^2 + x & \text{for } -1 \le x \le 0 \end{cases}$$

Assignment 10. *Partial Differential Equations*

Read Section 10.1, 10.2, 10.5, 10.6 (Nagle, Saff, Snider)

You should be able to do the following problems:

Section 10.2/Problem 15 - 21, Section 10.5/Problem 1 - 8, Section 10.6/Problem 1 - 4

Hand in the following problems:

1. If we find the Fourier series solution $u = u(x,t)$ for the equation

$$\frac{\partial u}{\partial t} = 3\frac{\partial^2 u}{\partial x^2}$$

where u is subject to the boundary conditions $u(0,t) = 0$ and $u(\pi,t) = 0$ then we obtain:

$$u(x,t) = \sum_{n=1}^{\infty} b_n e^{-3n^2 t} \sin nx$$

If $u(x,t)$ satisfies the initial condition $u(x,0) = \sin x \cos x$ then calculate the coefficients b_n.

2. Suppose we wish to find a solution $u = u(x,t)$ to the partial differential equation

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$$

where u satisfies the boundary conditions $\frac{\partial u}{\partial x}(0,t) = 0$ and $\frac{\partial u}{\partial x}(\pi,t) = 0$. Show how if we first find all nontrivial solutions of the form $X(x)T(t)$ and then form a Fourier series from them we can obtain the general solution of the equation.

Note: You are not given any initial conditions, so the coefficients of your final Fourier series solution will not be determined.

3. If we substitute $u = X(x)T(t)$ into the heat equation

$$\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$$

we obtain

$$\frac{T'(t)}{\alpha^2 T(t)} = \frac{X''(x)}{X(x)} = -\lambda^2$$

where λ is called the *separation constant.*

If u satisfies the boundary conditions $u(0,t) = 0$ and $\frac{\partial u}{\partial x}(\pi,t) = 0$, find the possible values of λ that lead to nontrivial solutions for u.

4. The following equation is called *Laplace's Equation*:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

where $u = u(x,y)$ is the solution. We can solve this equation, as usual, by substituting a trial solution of the form $u = X(x)Y(y)$ and solving for the functions $X(x)$ and $Y(y)$ that lead to nontrivial solutions. Find these functions if u satisfies the following boundary conditions:

$$u(0,y) = 0 \text{ and } u(\pi,y) = 0 \text{ for all } y \text{ between } 0 \text{ and } \pi$$

$$u(x,\pi) = 0 \text{ for all } x \text{ between } 0 \text{ and } \pi$$

Assignment 11. *Fourier Transforms*

1. Calculate the Fourier transform of both of the following two functions.

a.

$$u(x) = \begin{cases} \pi & \text{when } \; -4 < x < 4 \\ 0 & \text{otherwise} \end{cases}$$

b.

$$u(x) = \begin{cases} x\,e^{-x} & \text{when } \; x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

For the next three problems, use the method of Fourier Transforms to solve for $u(x, t)$ for the given partial differential equation.

2.

$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = 0 \qquad \text{where} \quad u(x, 0) = \frac{1}{1 + x^2}$$

3.

$$\frac{\partial u}{\partial t} + u = \frac{\partial u}{\partial x} \qquad \text{where} \quad u(x, 0) = e^{-x^2/4}$$

4. Use the method of Fourier transforms to find the function $u = u(x, t)$ that solves the following partial differential equation. Simplify your answer.

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} \qquad \text{where } u(x, 0) = 0 \quad \text{and} \quad \frac{\partial u}{\partial t}(x, 0) = xe^{-x^2/2}$$