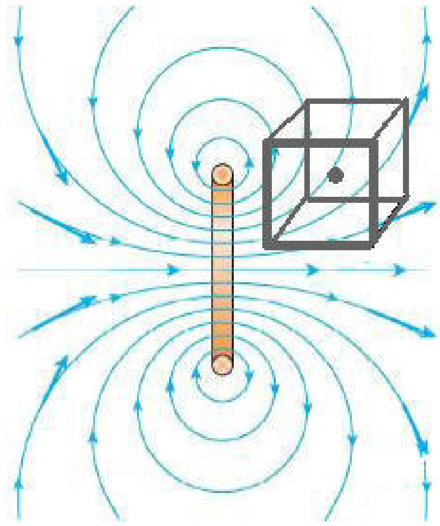
Gauss's Law for Electric Fields
(Differential Form)

Dr. Elliott Jacobs

$$\iint_S \vec{\mathbf{E}} \bullet \vec{\mathbf{n}}\, dS = 4\pi k q$$

$$\iint_S \vec{\mathbf{B}} \bullet \vec{\mathbf{n}}\, dS = 0$$

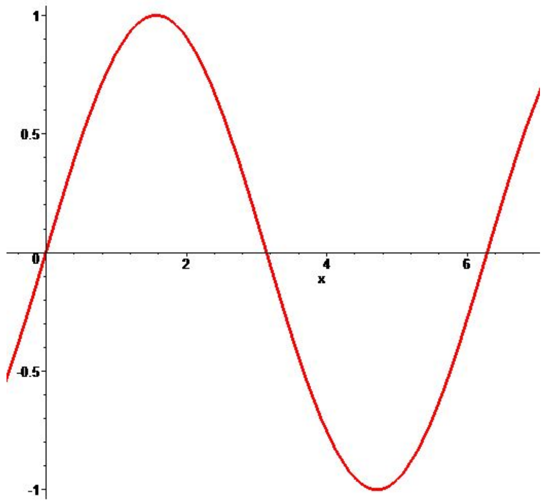
$$\nabla \bullet \vec{\mathbf{B}} = 0$$



Can an integral equal 0?

$$\int_a^b f(x)\,dx = 0$$

$$\int_0^{2\pi} \sin x \, dx = 0$$



Can an integral equal 0 for *every* interval $a \leq x \leq b$?

$$\int_a^b f(x)\, dx = 0$$

Can an integral equal 0 for *every* interval $a \le x \le b$?

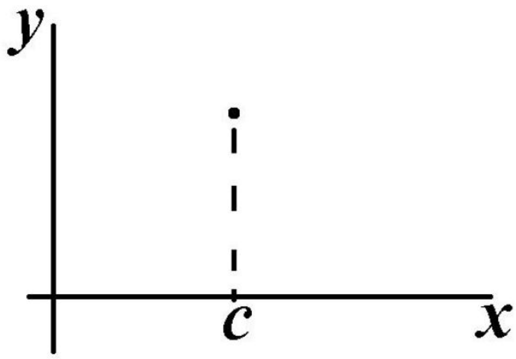
$$\int_a^b f(x)\,dx = 0$$

This will certainly happen if $f(x) = 0$ for all x

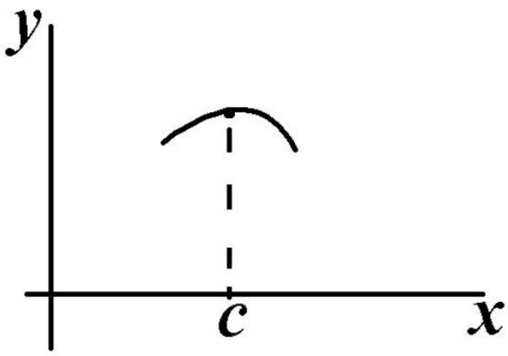
$$\int_a^b 0\,dx = 0$$

Theorem:
If $f(x)$ is continuous for all x and $\int_a^b f(x)\,dx = 0$ for every interval $[a,\ b]$ then $f(x) = 0$ for all x

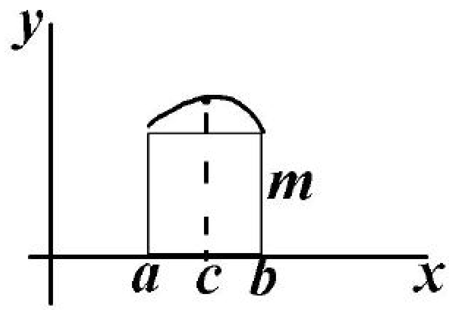
Suppose there were one point $x = c$ where $f(c) > 0$



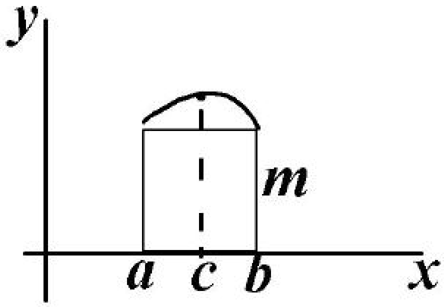
If f is a continuous function then $f(x) > 0$ for all x in some interval around c.



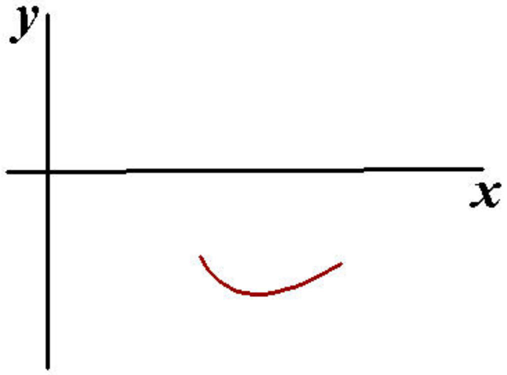
$$\int_a^b f(x)\,dx \geq m(b-a) > 0$$



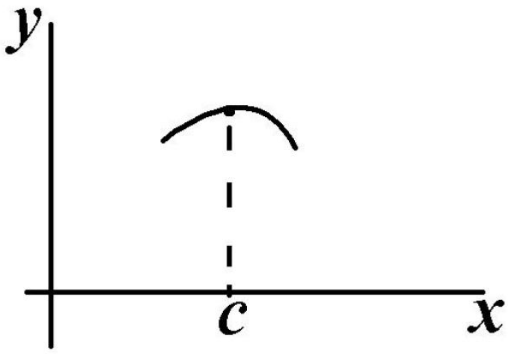
This shows that if f is a continuous function and $f(x) > 0$ at some x value, then $\int_a^b f(x)\,dx > 0$ for some interval $[a,\ b]$ surrounding this x value.



Similarly, if f is continuous and $f(x) < 0$ at some x value, then $\int_a^b f(x)\,dx < 0$ for some interval around this x value.



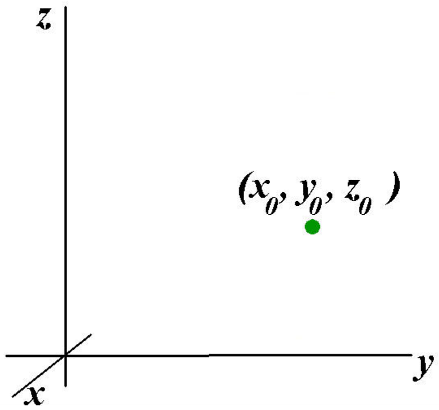
So, if f is a continuous function with $f(x) \neq 0$ for some x, then there will be some interval $[a,\ b]$ for which $\int_a^b f(x)\,dx \neq 0$



This implies that if $\int_a^b f(x)\,dx = 0$ for *every* interval, then $f(x) = 0$ for every x.

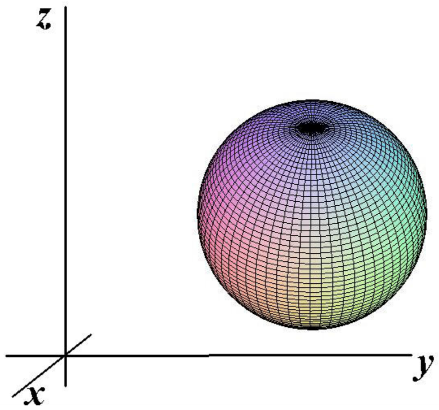
$$f = f(x, y, z) \qquad \text{(continuous)}$$

Supposed f is continuous for all x, y and z and there were some point at which the function is nonzero.



$$f = f(x, y, z) \qquad \text{(continuous)}$$

There would have to be some sphere V around this point for which $\iiint_V f(x, y, z)\, dV \neq 0$

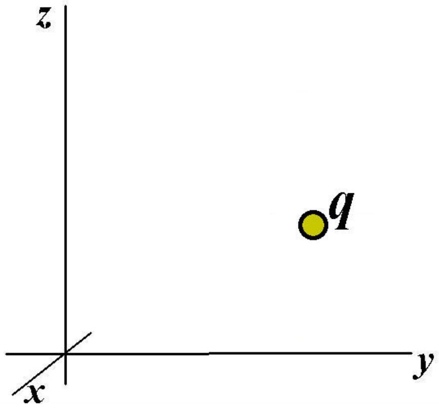


$$f = f(x, y, z) \qquad \text{(continuous)}$$

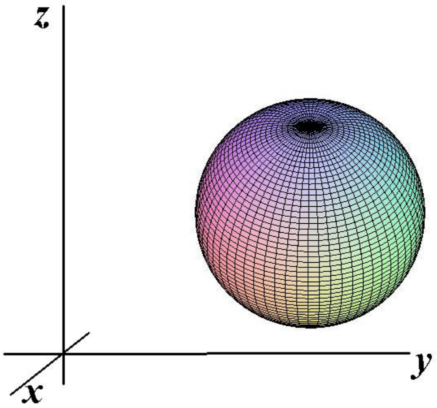
There would have to be some sphere V around this point for which $\iiint_V f(x, y, z)\, dV \neq 0$

This implies that if $\iiint_V f(x, y, z)\, dV = 0$ for all three dimensional regions V, then $f(x, y, z) = 0$ at all points

$$\iint_S \vec{\mathbf{E}} \bullet \vec{\mathbf{n}}\, dS = 4\pi k q$$



$$\iint_S \vec{\mathbf{E}} \bullet \vec{\mathbf{n}}\, dS = 4\pi k q = 4\pi k \iiint_V \rho\, dV$$



$$\iint_S \vec{\mathbf{E}} \bullet \vec{\mathbf{n}}\, dS = 4\pi k q = 4\pi k \iiint_V \rho\, dV$$

$$\iiint_V \nabla \bullet \vec{\mathbf{E}}\, dV = 4\pi k \iiint_V \rho\, dV$$

$$\iint_S \vec{\mathbf{E}} \bullet \vec{\mathbf{n}}\, dS = 4\pi k q = 4\pi k \iiint_V \rho\, dV$$

$$\iiint_V \nabla \bullet \vec{\mathbf{E}}\, dV = 4\pi k \iiint_V \rho\, dV$$

$$\iiint_V \left(\nabla \bullet \vec{\mathbf{E}} - 4\pi k \rho\right) dV = 0$$

$$\nabla \bullet \vec{\mathbf{E}} - 4\pi k \rho = 0$$

$$\nabla \bullet \vec{\mathbf{E}} = 4\pi k \rho$$

(at all points)

Gauss's Law for Electric Fields

Integral form:

$$\iint_S \vec{\mathbf{E}} \bullet \vec{\mathbf{n}}\, dS = 4\pi k q$$

Differential form:

$$\nabla \bullet \vec{\mathbf{E}} = 4\pi k \rho$$

$$\iint_S \vec{\mathbf{E}} \bullet \vec{\mathbf{n}}\, dS = 4\pi k q$$

$$\nabla \bullet \vec{\mathbf{E}} = 4\pi k \rho$$

Let $\epsilon_0 = \frac{1}{4\pi k}$

$$\iint_S \vec{\mathbf{E}} \bullet \vec{\mathbf{n}}\, dS = \frac{q}{\epsilon_0}$$

$$\nabla \bullet \vec{\mathbf{E}} = \frac{\rho}{\epsilon_0}$$